Object Adaptive Imaging of Dynamic Scenery: Application to Cardiac MRI

Image Analysis and Understanding Data from Scientific Experiments Workshop

Los Alamos, December 2nd -6th

Nitin Aggarwal, Yoram Bresler

Coordinated Sciences Labs and ECE department

University of Illinois, Urbana-Champaign

Acknowledgements :

Work supported by NIH grant R21HL62336-02 and NSF grant BES02-01876

Travel to workshop supported by LANL

Outline

1. Dynamic imaging and Time-Sequential Sampling constraint
2. Time-Sequential Sampling (TSS) Theory
3. Application to cardiac MRI
    - Adaptive MR imaging
    - Time-warped models

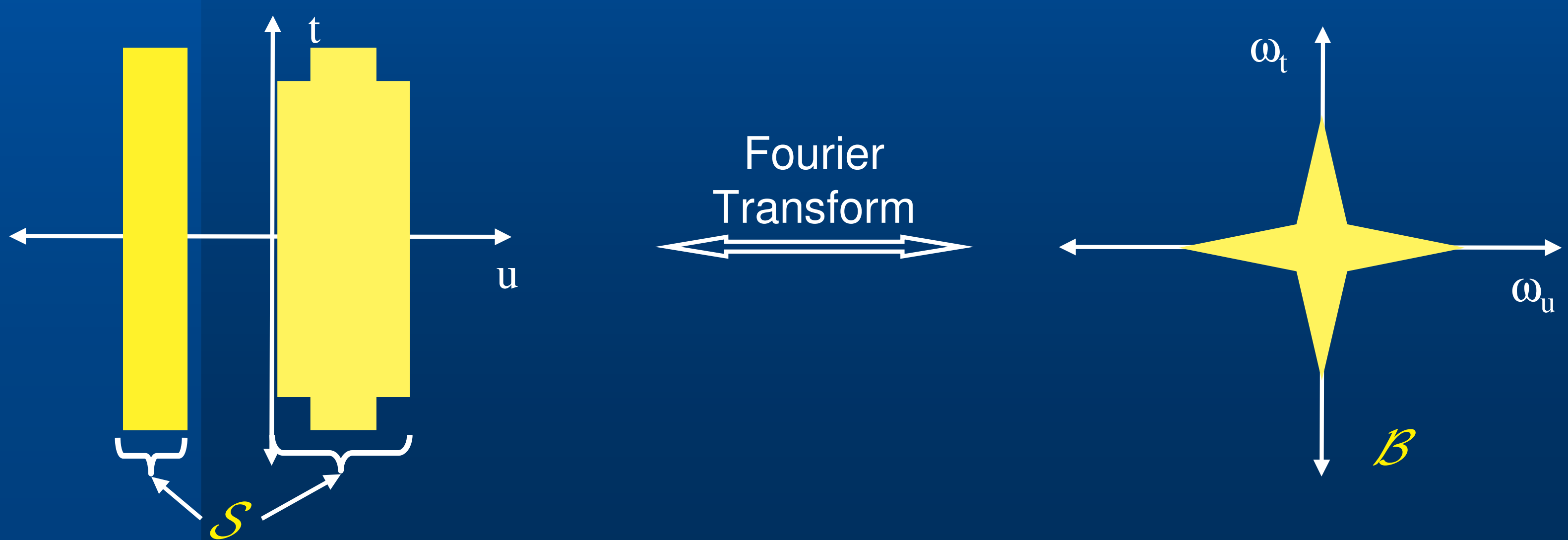
Dynamic Imaging

Problem:

Given a class $\mathcal{M} \subset \mathbb{R}^n$ of time-varying signals $g(\boldsymbol{u},t)$ with :

1. Spatial support : $$\mathcal{S} \triangleq \bigcup_t \mathrm{supp}\{g(\bullet,t)\}$$
2. Spatio-temporal Spectral Support (essential) : $$\mathcal{B} \triangleq \mathrm{supp}\{\int g(\mathbf{u},t)\, e^{-j(\boldsymbol{\omega}_{\mathbf{u}}\mathbf{u}+\omega_t t)}\, d\mathbf{u}dt\}$$

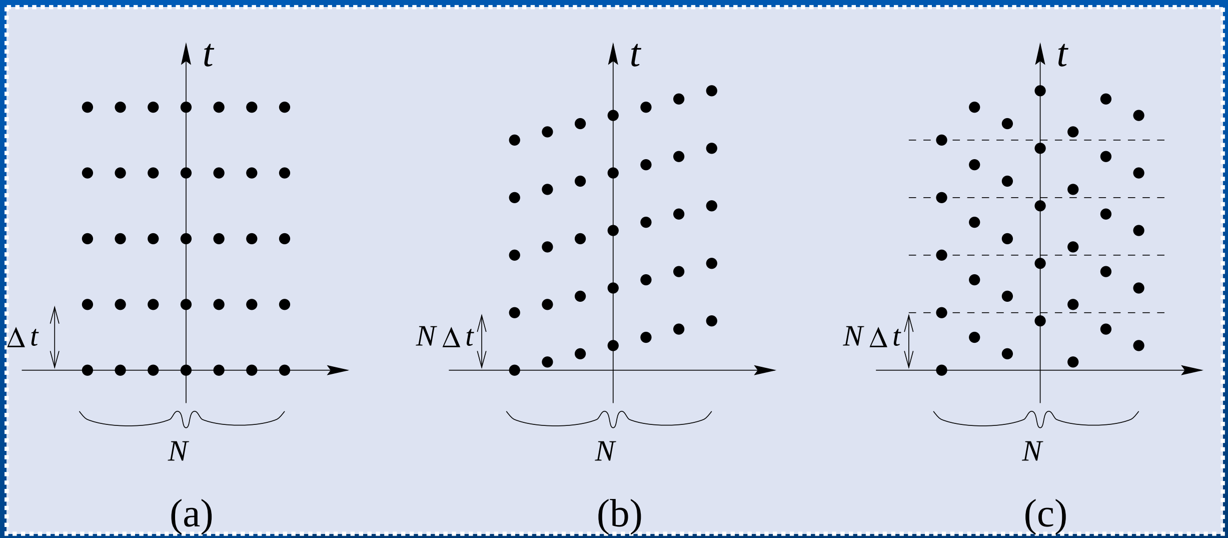
Find a sampling schedule $\Psi=\{\boldsymbol{u}_l,t_l\}_l$ so that $g(\boldsymbol{u},t)$ is recoverable from the corresponding samples.



Time-Sequential Sampling Constraint

Definition:

A sampling schedule $\Psi=\{\boldsymbol{u}_l, t_l\}_l$ is time-sequential if only <u>one</u> point in u is acquired at any given time i.e. $t_n \neq t_m$ for $n \neq m$.



a) Instantaneous sampling

b) Progressive TSS

c) Scrambled TSS

Applications:

Any imaging system with mechanical or electrical scanning like

- Radar or acoustic imaging
- Cardiac Magnetic resonance imaging (MRI)

TSS Schedule Design

Problems :

- Conditions on sampling rate ?
- Design of sampling pattern ?
- Reconstruction from acquired samples ?

Properties :

- Aliasing in TSS determined not only by density of points (sampling rate) in Ψ, but also by the order in which points are visited [Allebach,1987]
- Order in which sample points are visited need not be determined by their adjacency (in $\mathcal{S}$)
- Optimization of Ψ is a very difficult combinatorial problem. Need to explore 256! possible orderings for 256 sample locations.

TSS Schedule Design

Solution :

- Unified TSS theory presented in Willis & Bresler, 1997
- Key idea :

  Consider TSS schedules that lie on a lattice (a periodic, regular set of points) in ($\boldsymbol{u}$, t) space.

Result :

- Sampling pattern design through constrained geometric packing of $\mathcal{B}$, $\mathcal{S}$ …
- Reconstruction of g(u,t) through linear filtering
- Bounds on achievable TS sampling rates

- In practice, sampling rate reduced by large factor compared to conventional sampling schemes.

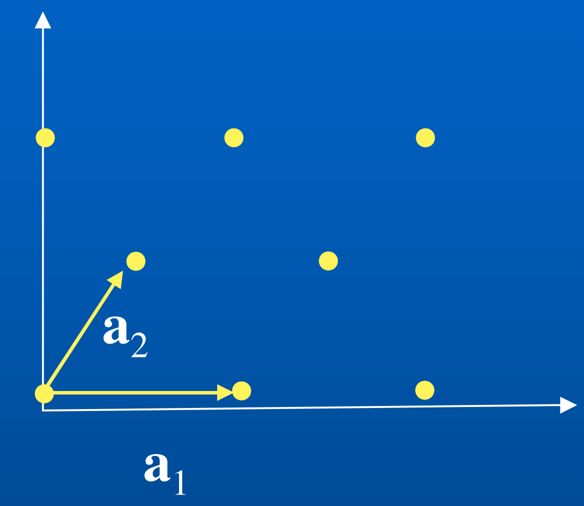
Time-Sequential Sampling Theory

Lattice Theory

Lattice : A lattice Λ_A is the set :

$$\Lambda_{\mathbf{A}} = \{\sum_{i=1}^{n} m_i \mathbf{a}_i : m_i \in \mathbb{Z}\}$$

where $\mathbf{a}_i$ (in $\mathbb{R}^n$) are linearly independent



Basis Matrix : A basis matrix of Λ_A is $A = [a_1, a_{2, \ldots}]$

Result : Every (rational) lattice has a basis matrix of the form $\mathbf{A} = \begin{pmatrix} B & s \\ 0 & T_R \end{pmatrix}$

Polar Lattice : The polar lattice of a lattice Λ_A is a lattice Λ_{A*} with basis matrix $A^* = A^{-T}$

Multidimensional Sampling Theory

Result :

If a signal is sampled on lattice Λ_A then it's spectrum is replicated in the frequency domain on the polar lattice Λ_{A*}

$$\mathrm{DTFT}\{g(\mathbf{Am})\}(\bullet) = \frac{1}{|\det(\mathbf{A})|} \sum_{\mathbf{k}\in\mathbb{Z}^2} \mathrm{FT}\{g\}(\bullet - \mathbf{A}^{*}\mathbf{k})$$

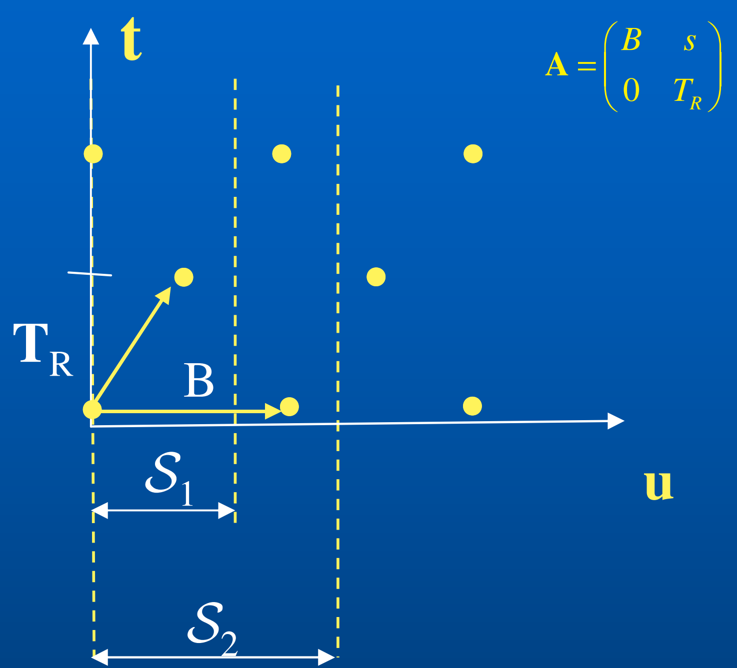
Therefore the signal *g* can be recovered from it's samples iff the replicas do not overlap

- The lattice Λ_{A*} *packs* the spectral support $\mathcal{B}$ of g

Notation : $\Lambda_A \,\varepsilon\, \mathcal{R}(\mathcal{B})$

Time-Sequential Sampling on a Lattice

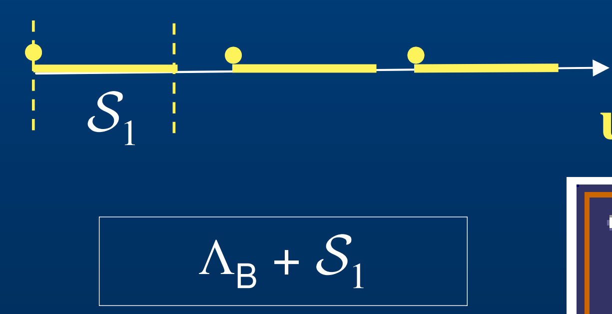
If sample points (u_n, t_n) are chosen to lie on a lattice Λ_A, in general, the time-sequential constraint will not be met



However there may be *only one sample point in* $\mathcal{S}$ at any given sampling time instant i.e Λ_A is time-sequential w.r.t. $\mathcal{S}$

Notation : $\Lambda_A \; \varepsilon \; \mathcal{T}(\mathcal{S})$

Result : $\Lambda_A \; \varepsilon \; \mathcal{T}(\mathcal{S})$ if lattice spanned by B (i.e. Λ_B), packs $\mathcal{S}$



Temporal uniformity constraint

Definition :

Temporally uniform lattice :

A lattice Λ_A is temporally uniform w.r.t support $\mathcal{S}$, if the time instants at which samples are obtained are uniformly spaced

Notation : $\Lambda_A \,\varepsilon\, \mathcal{U}(\mathcal{S})$

Result :

If $(\Lambda_B, \mathcal{S})$ tiles $\mathbb{R}$ ($\mathbb{R}^{n-1}$ in general) i.e. replicas of $\mathcal{S}$ cover $\mathbb{R}$ without gaps or overlaps, then the lattice $\Lambda_A \,\varepsilon\, \mathcal{T}(\mathcal{S}) \cap \mathcal{U}(\mathcal{S})$

Sampling Schedule Design

Find : A lattice sampling schedule $\Lambda_A = \{u_n, t_n\}_n$ such that :

1. Any signal in $\mathcal{M}$ can be recovered from the samples
2. Schedule is time-sequential and temporally uniform w.r.t $\mathcal{S}$ (with time period T_R),
3. T_R is maximized

Solution:

$$\underset{\mathbf{A}=\begin{pmatrix} B & s \\ 0 & T_R \end{pmatrix};\Lambda_{\mathbf{A}} \in \mathcal{R}(\mathcal{B}) \cap \mathcal{T}(\mathcal{S}) \cap \mathcal{U}(\mathcal{S})}{\arg\max} T_R$$

Solution computed by searching for lattices subject to the "dual" packing constraints :

1. Λ_{A*} packs the spectral support $\mathcal{B}$
2. $(\Lambda_B, \mathcal{S})$ tiles $\mathbb{R}$

Reconstruction and Results

Reconstruction method :

Filter the samples with filter with frequency response $H(\omega_u, \omega_t) = \chi_{\mathcal{B}}(\omega_u, \omega_t)$

Performance bounds for TSS :

$$T_R(\Lambda, \mathcal{S}) \leq \frac{1}{d(\Lambda_{crit}(\mathcal{B})) \cdot volume(\mathcal{S})} \leq \frac{1}{volume(\mathcal{B}) \cdot volume(\mathcal{S})}$$

$$\text{Speed Gain factor} = \mathrm{G} \triangleq \frac{\mathrm{T_{opt}}}{\mathrm{T_{prog}}} \leq \frac{volume(bounding\ box(\mathcal{B}))}{volume(\mathcal{B})}$$

Asymptotically and in practice:

- Bounds achievable
- No penalty for restriction to lattice patterns !
- No penalty for time-sequential constraint !

Adaptive Magnetic Resonance Imaging

Applications and challenges

Applications :

1. Cardiac MRI :
    a. Visualization of coronary arteries
    b. Functional assessment of the ventricles
    c. Myocardial flow, perfusion and viability
    d. Vascular disease and tissue characterization
    e. Myocardial dynamics
2. Functional MRI
3. Interventional MRI

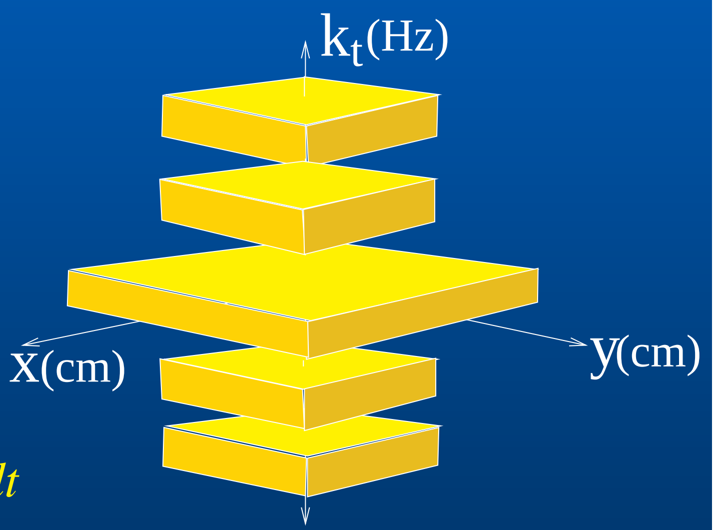
Challenges :

- MR coronary angiography : Current resolution limited to 1mm
- Cardiac imaging without breatholding
- Plaque characterization in coronary angiography : requires spectral imaging, in addition to high spatio-temporal resolution

Cardiac Image models

Characteristics :

- The (highly) dynamic portion of the object (i.e heart) is spatially localized within the field-of-view.
- The cardiac motion is quasi-periodic



Model :

Object to be imaged : $I(x, y, t)$

Spatio-temporal spectrum : $I(x, y, k_t) = \int I(x, y, t) e^{-ik_t t} dt$

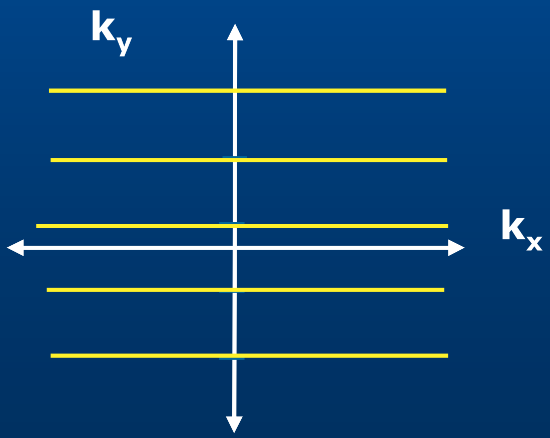
Spatio-Temporal Spectral support : $\mathrm{supp}\{I(x, y, k_t)\}$

MR Imaging

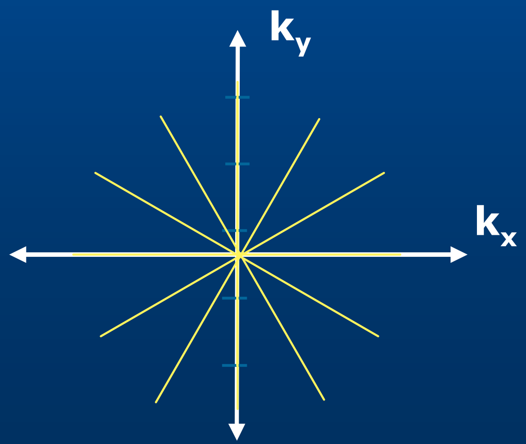
Imaging Equation :

$$s(k(t)) = \int_{FOV} I(r,t)e^{-ik(t)\cdot r}\,dr \qquad t \ll T_2$$

- Signal is (spatial) Fourier transform of object *I (r,t)* evaluated at a certain frequency determined by trajectory of *k(t)*
- Trajectory of *k(t)* determined by applied magnetic gradient field
- Length of trajectory cannot be too large



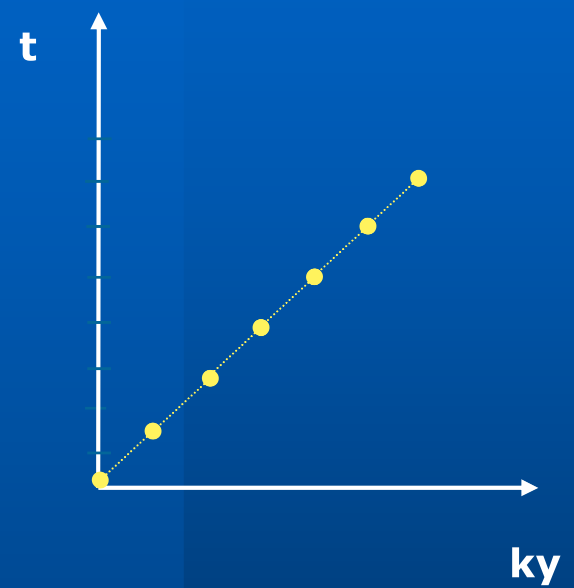
Rectangular Sampling



Radial Sampling

The k-t space perspective

The (k_y, t) trajectory …



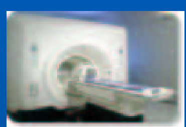
Time-Sequential Sampling Constraint :

Data at *only one* k_y can be acquired at a given time instant

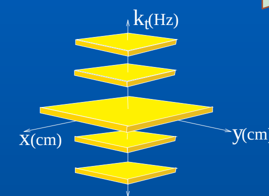
Adaptive Imaging Scheme

Pre-Imaging

Pilot-scan Acquisition → Spectral Analysis

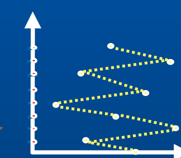


Spatio-temporal-spectral support →

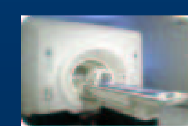


Adaptation of Sampling Pattern

Sampling pattern ↓

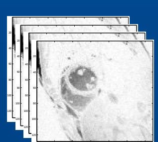


Imaging Acquisition



← Image Reconstruction

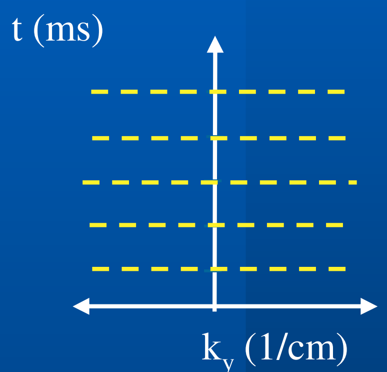
← cine



A. Pre-imaging Acquisition

Aim : To estimate the support of $I(k_x,y,k_t)$

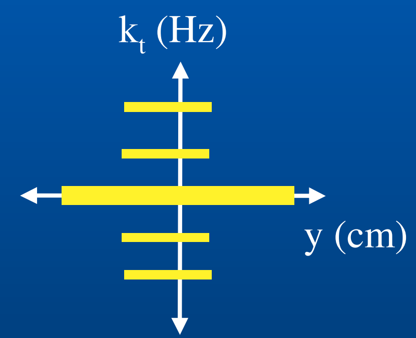
Method : For each k_x …



$I(k_x,k_y,t)$

k_x – phase-encode
k_y - readout

Spectral Analysis



Support of $I(k_x,y,k_t)$

$$\mathcal{B} = \bigcup_{k_x} supp\{I(k_x, y, k_t)\}$$

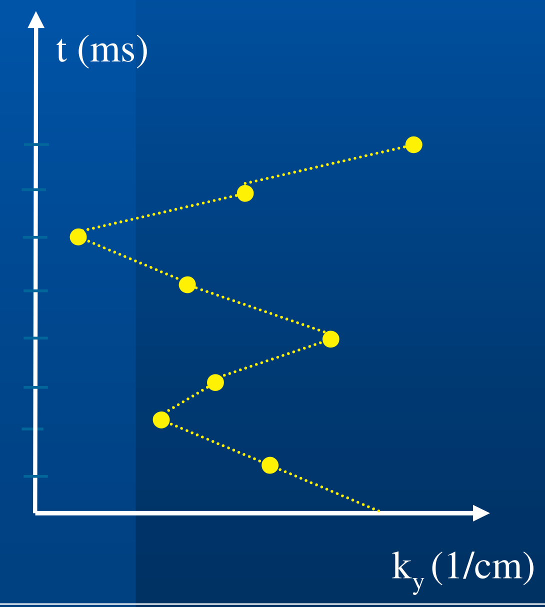
B. Adaptation of Imaging

Given :

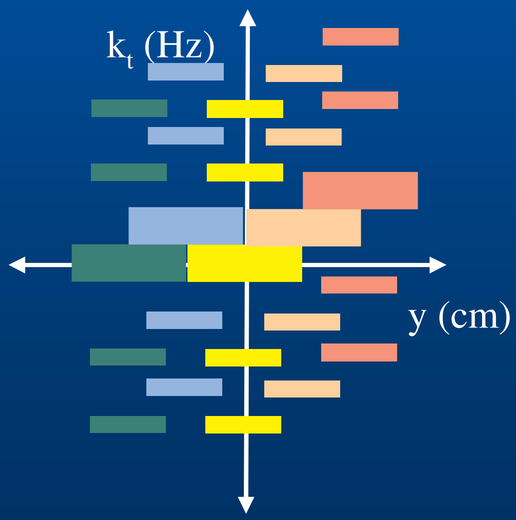
- Spatio-temporal spectral support $\mathcal{B} \subset \mathbb{R}^2$ (from pre-imaging)
- 'Spatial' support set $\mathcal{S}$=[- $k_{y,max}$, $k_{y,max}$] determined by desired spatial resolution ($k_{y,max}$)

Find :

Minimum rate, temporally uniform, TS sampling schedule $\Lambda_A=\{k_y(n),nT_R\}$



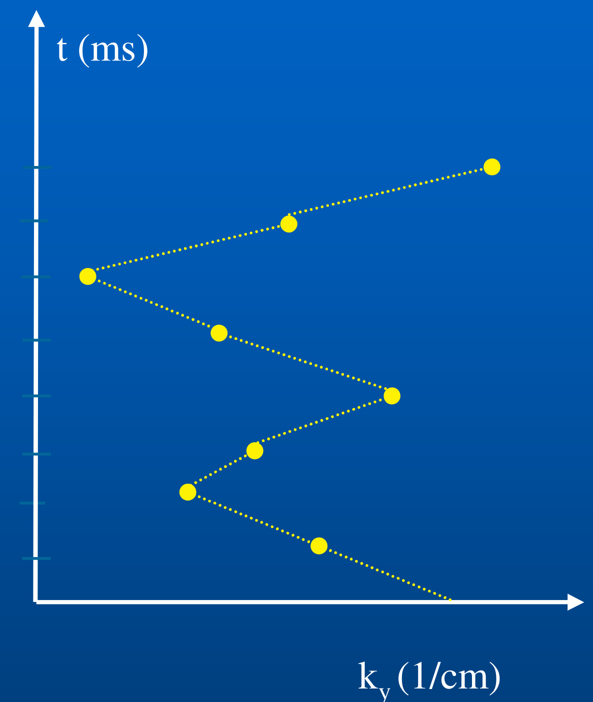
Lattice Sampling Schedule



Lattice packing of $\mathcal{B}$

C. Imaging Acquisition

Acquire data according to the sampling schedule $\{k_y(n), nT_R\}$



k_x – readout

k_y – phase-encode

D. Reconstruction

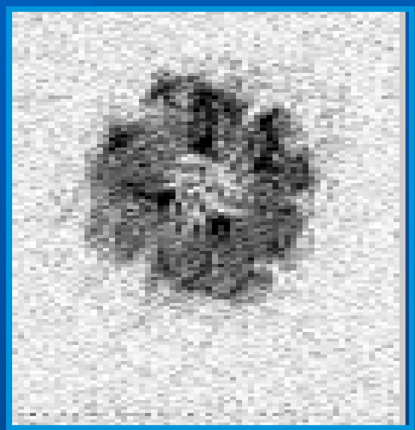
Filter acquired samples with filter with impulse response

$$H(y,k_t) = \chi_B(y,k_t)$$

Results (Experimental Parameters)

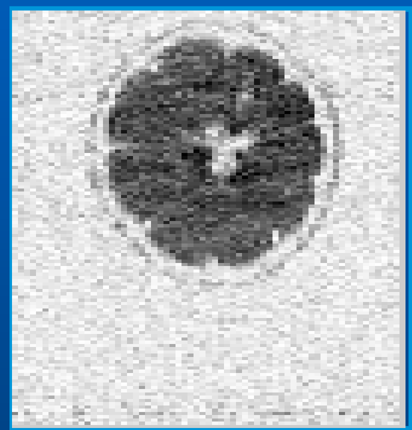
- Equipment: VARIAN/SISCO 4.7T imaging spectrometer.
- MRI Parameters: FLASH pulse sequence with flip angle 9°, TR=10ms (optimally chosen for Time-sequential sampling), total acquisition time 20s.
- Object: lemon moved periodically by a motor.
- Motion: Periodic with period approximately 1.5 Sec and involves :
  - Translation in readout direction
  - Translation in PE direction
  - In plane rotation
  - Out of plane rotation and translation

Simulation Result



Progressive sampling

(96 frames, 25 fps)



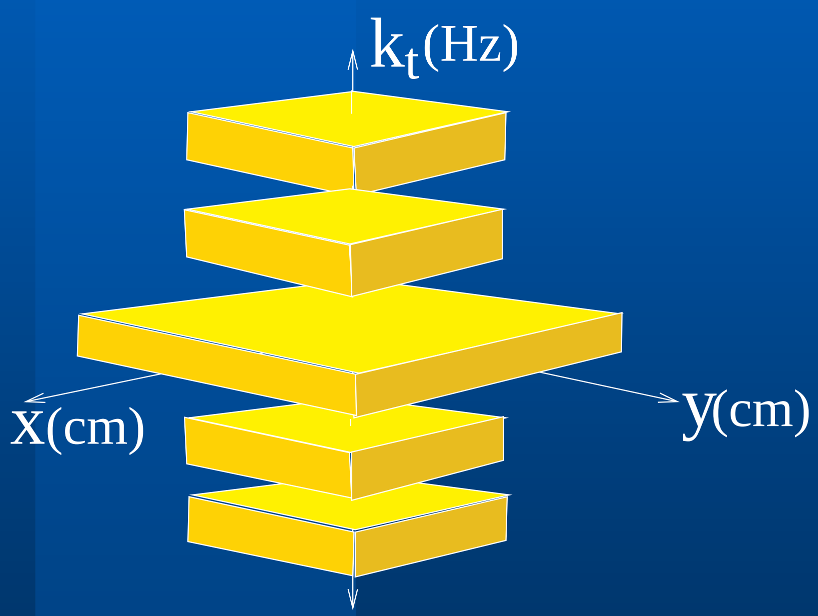
TS Lattice Sampling

(96 frames, 25 fps)

Time-warping:
Modeling Cardiac Aperiodicity

Role of Spectral model

Spatio-Temporal spectral model :



- Regions with significant temporal variation spatially localized
- Cardiac motion is quasi-periodic

Ł Spectral bands

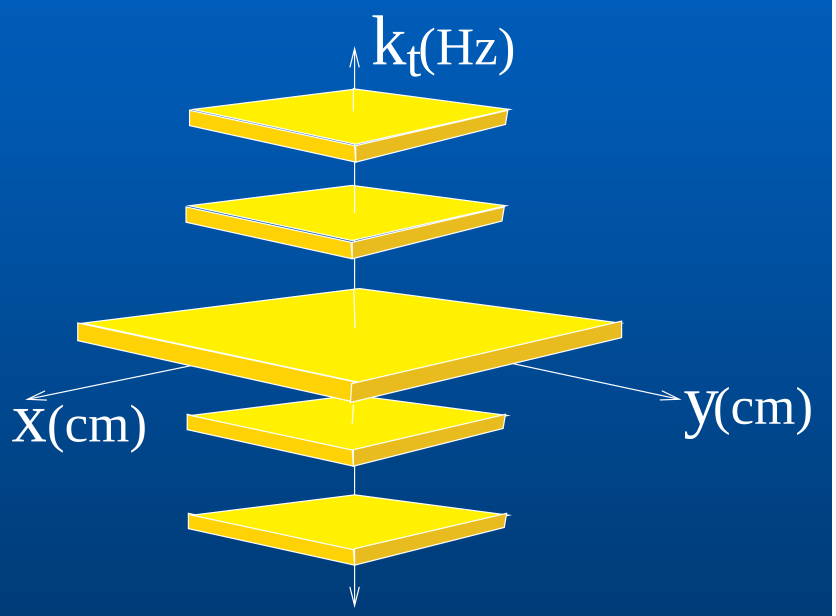
- Spectral bands broadened due to aperiodicity of dynamics

Ł Higher sampling rate required

Role of Time-Warp

Time-Warp Model :

$$I(x, y, t) = G(x, y, \Phi(t))$$



I(x,y,t) : Time-varying cardiac image

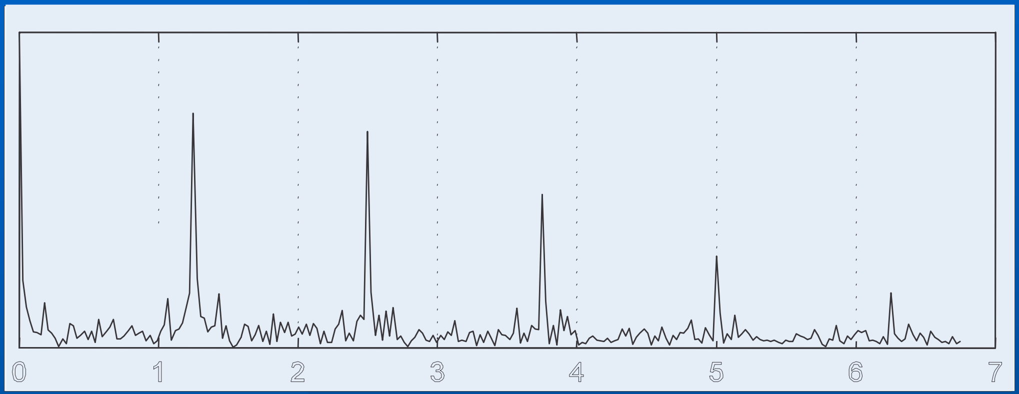
G(x,y,t) : Idealized time-varying cardiac image

Φ(t) : Time-warp

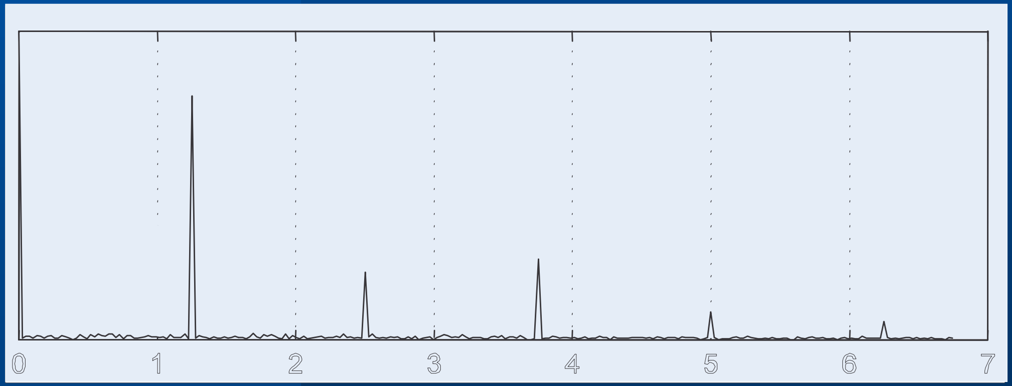
Explicitly accounting for time-warp narrows spectral support

Ł Lowers sampling rate required

Role of Time-Warp



Temporal Spectrum
(before dewarping)



Temporal Spectrum (after dewarping)

Models :

$$I(x,y,t) = G(x,y,\Phi(t))$$

- Time warp $\Phi(t)$: Assumed to be monotonic, slowly varying
- Idealized cardiac cine G(x,y,t) :

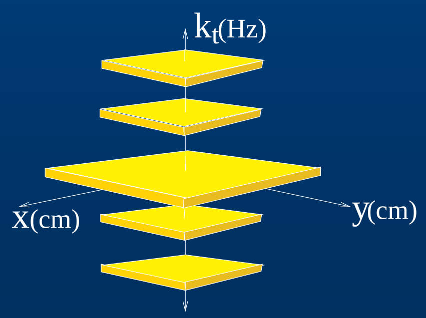
a. Time-Warped Harmonic Model :

$$G(x,y,t) = \sum_{m=-M}^{M} \alpha_m(x,y) \cdot e^{j2\pi m f_0 t}$$

§ α_m outside modeled spectral support = 0

b. Time-warped banded spectral model :

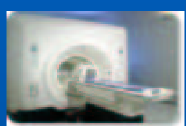
§ G(x,y,t) has a narrow banded spectral support



Imaging Scheme

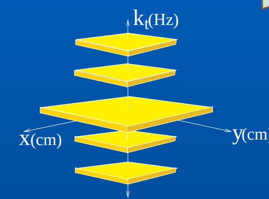
Pre-Imaging

Pilot scan Acquisition



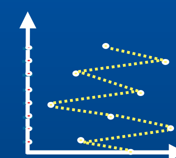
Spectral Analysis

Spatio-temporal-spectral support



Adaptation of Sampling Pattern

Sampling pattern



Imaging Acquisition

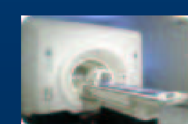
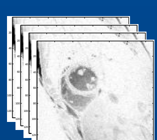


Image Reconstruction

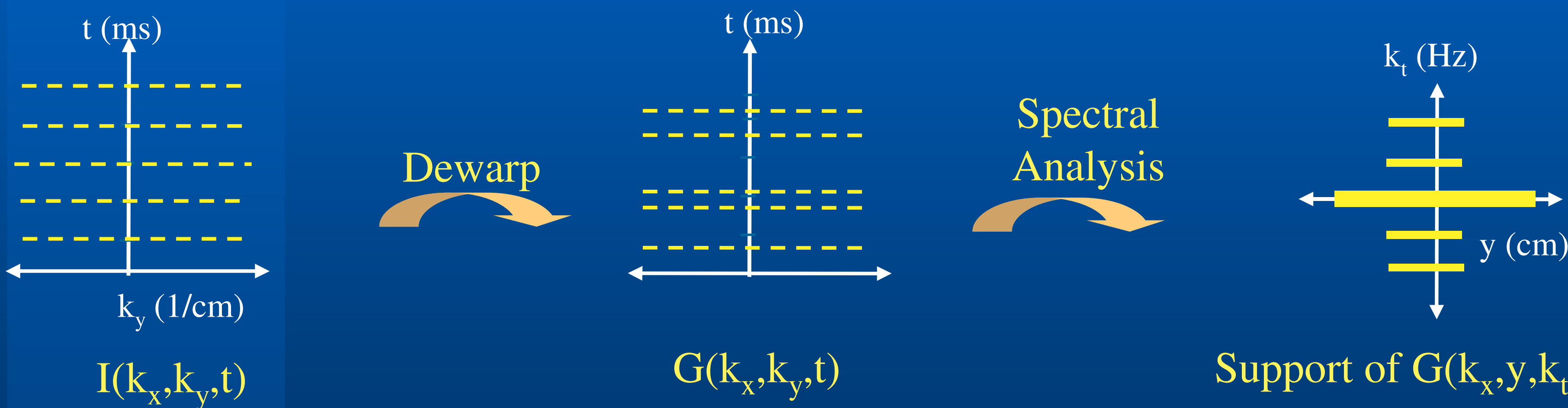
cine



A. Pre-imaging Acquisition

Aim : To estimate the support of $G(k_x,y,k_t)$

Method : For each k_x …

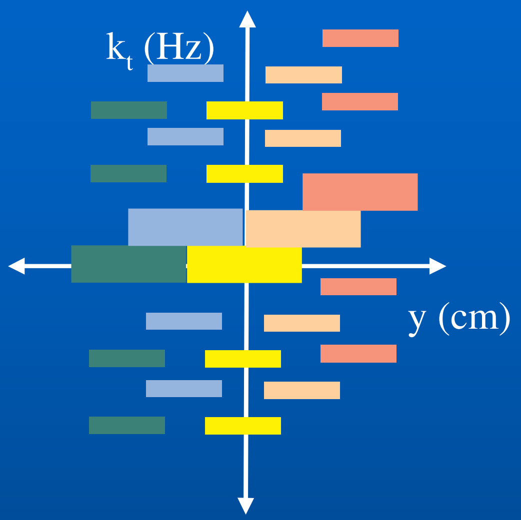


k_x – phase-encode

k_y - readout

B. Adaptation of Imaging

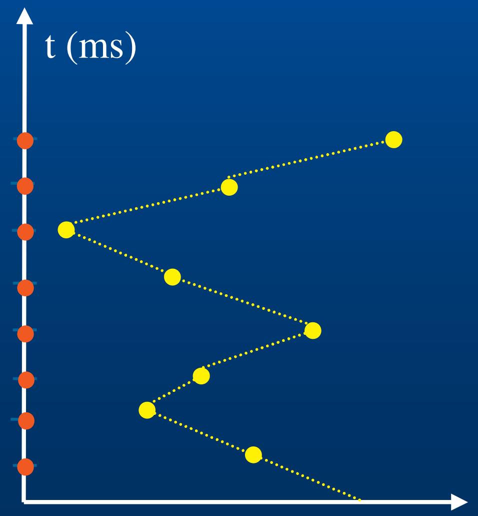
Sampling lattice adapted to spatio-tempral spectral support of $G(k_x,y,k_t)$



C. Imaging acquisition

Use 2-echo MR acquisition :

- First echo : k-space lines for cine reconstruction
- Second echo : navigator data for time-warp estimation



D. Cine Reconstruction

a. Time-Warped Harmonic Model:

$$I(x,k_y,t) = \sum_{m=-M}^{M} \alpha_m(x,k_y) \cdot e^{j2\pi m f_0 \Phi(t)}$$

- Use navigator data to estimate :
  1. Time-warp $\Phi(t)$
     (dynamic programming)
  2. Fundamental frequency f_0
     (nonlinear least-squares with nonuniform samples)

- Use Imaging data to estimate α_m
  (linear least-squares)

D. Cine Reconstruction …

b. **Time-Warped Banded Spectral Model:**

- Use navigator data to estimate time-warp $\Phi(t)$
- Dewarp sampling instants nT_R to $t_n = \Phi^{-1}(nT_R)$
- $G(k_x,k_y,t)$ is available at non-uniform (warped) time-instants t_n

- Γ = Set of possible reconstructions …
  - Consistent with acquired data
  - Consistent with modeled spectral support
  - Energy bounded by some constant E
- $G^*(x,y,t)$ chosen such that :

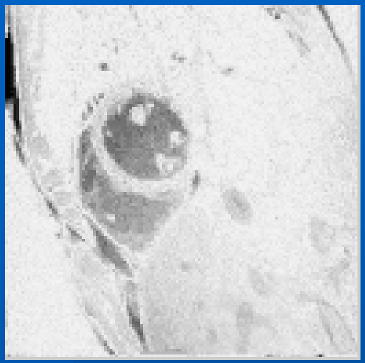
$$G^*(x,y,t) = \arg\min_{G(x,y,t)\in\Gamma} \sup_{H(x,y,t)\in\Gamma} \left\| G(x,y,t) - H(x,y,t) \right\|$$

Simulation Setup

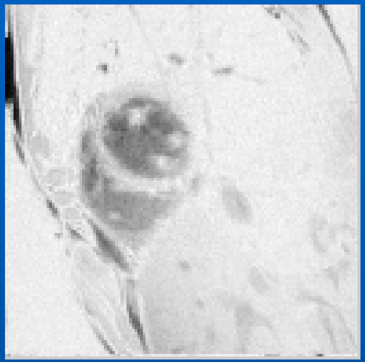
- Cardiac MRI cine used as a heart phantom
- Phantom driven aperiodically using real ECG data
- Phantom used to simulate Pre-Imaging and Imaging data

- Original cine data :
  - Ø 80 frames; 128x128 pixels
  - Ø Segmented true FISP; 8 receiver channels; 3 echos/segment
  - Ø T_R = 2.6 ms; T_{acq} =16 s
- Acquisition using proposed scheme :
  - Ø Optimal T_R = 8.9 ms
    - Ł 20-fold slower sampling

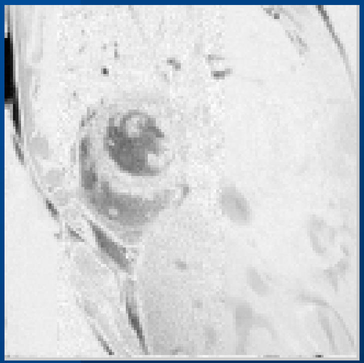
Simulation Result



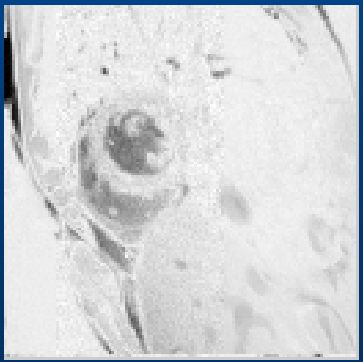
Original cine



Conventional reconstruction



Time-warped Harmonic Reconstruction



Time-warped banded spectral reconstruction

Discussion

- Improved adaptive data sampling and reconstruction methods for imaging time-varying objects under TS constraint
- The whole process of spectral analysis, adaptation of acquisition, reconstruction can be automated in many applications
- Application to cardiac MRI allows for 20-fold slower sampling than conventional methods
- Explicit modeling of cardiac aperiodicity reduces data redundancy and allows for even slower acquisition